

# $\phi\phi$ Back-to-Back Correlations in Finite Expanding Systems

Sandra S. Padula\*, Y. Hama<sup>†</sup>, G. Krein\*, P. K. Panda\*\* and T. Csörgő<sup>‡</sup>

\**Inst. de Física Teórica, UNESP - Rua Pamplona 145, 01405-900 São Paulo, SP - Brazil*

<sup>†</sup>*Instituto de Física, USP, Caixa Postal 66318, 05389-970 São Paulo, SP - Brazil*

\*\**Depto. de Física-CFM, UFSC - C. P. 476, 88040-900 Florianópolis, SC - Brazil*

<sup>‡</sup>*MTA KFKI RMKI, H - 1525 Budapest 114, P.O. Box 49, Hungary*

**Abstract.** Back-to-Back Correlations (BBC) of particle-antiparticle pairs are predicted to appear if hot and dense hadronic matter is formed in high energy nucleus-nucleus collisions. The BBC are related to in-medium mass-modification and squeezing of the quanta involved. Although the suppression of finite emission times were already known, the effects of finite system sizes and of collective phenomena had not been studied yet. Thus, for testing the survival and magnitude of the effect in more realistic situations, we study the BBC when mass-modification occurs in a finite sized, thermalized medium, considering a non-relativistically expanding fireball with finite emission time, and evaluating the width of the back-to-back correlation function. We show that the BBC signal indeed survives the expansion and flow effects, with sufficient magnitude to be observed at RHIC.

**Keywords:** Modified mass in hot-dense medium, squeezed states, particle-antiparticle correlation

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In the late 1990's the *Back-to-Back Correlations (BBC)* between boson-antiboson pairs were shown to exist if the particles masses were modified in a hot and dense medium[1]. Not much longer after that, it was also shown that a similar BBC existed between fermion-antifermion pairs with medium-modified masses[2]. A similar formalism is applicable to both BBC cases, related to the Bogoliubov-Valatin transformations of in-medium and asymptotic operators. Both the bosonic (bBBC) and the fermionic (fBBC) Back-to-Back Correlations are positive and have unlimited magnitude, thus differing from the identical-particle correlations, also known as HBT (Hanbury Brown & Twiss) correlations, which are limited for both cases, being negative in the fermionic sector. BBC were expected to be significant for  $p_T < 2$  GeV/c. Nevertheless, already in the Ref.[1], it has been shown that the duration of the emission process significantly suppresses its magnitude. The effects of finite system sizes and of collective phenomena were not known, which motivated us to investigate their consequences. Some preliminary results will be discussed here and illustrated in some particular cases.

## 0.1. Infinite Homogeneous Medium

In our analysis, we assume the validity of local thermalization and hydrodynamics, from the beginning up to the system freeze-out, as well as a short duration of the particle emission. We also consider that the effective in-medium Hamiltonian can be written as

$$H = H_0 - \int d\mathbf{x} d\mathbf{y} \phi(\mathbf{x}) \delta M^2(\mathbf{x} - \mathbf{y}) \phi(\mathbf{y}), \quad (1)$$

where

$$H_0 = \int d\mathbf{x} (\dot{\phi}^2 + |\nabla\phi|^2 + m^2\phi^2) \quad (2)$$

is the asymptotic (free) Hamiltonian in the matter rest frame, and the second term in Eq. (1) describes the medium modifications. The scalar field  $\phi$  represents quasi-particles propagating with momentum-dependent medium-modified mass  $m_*$ , related to the vacuum mass,  $m$ , by

$$m_*^2(|\mathbf{k}|) = m^2 - \delta M^2(|\mathbf{k}|).$$

This implies that the dispersion relations in the vacuum and in-medium are given, respectively, by  $\omega_k^2 = m^2 + \mathbf{k}^2$  and  $\Omega_k^2 = m_*^2 + \mathbf{k}^2 = \omega_k^2 - \delta M^2(|\mathbf{k}|)$ , where  $\Omega$  is the frequency of the in-medium mode with momentum  $\mathbf{k}$ .

The annihilation (creation) operator,  $b$  ( $b^\dagger$ ), for the in-medium, thermalized quasi-particles is related to the annihilation (creation) operator of the asymptotic, observed quanta with momentum  $k^\mu = (\omega_k, \mathbf{k})$ ,  $a$  ( $a^\dagger$ ), by the Bogoliubov-Valatin transformation

$$a_k^\dagger = c_k^* b_k^\dagger + s_{-k} b_{-k}; \quad a_k = c_k b_k + s_{-k}^* b_{-k}^\dagger, \quad (3)$$

where  $c_k = \cosh(f_k)$  and  $s_k = \sinh(f_k)$ ;  $f_k = \frac{1}{2} \log(\frac{\omega_k}{\Omega_k})$  is the so-called *squeezing parameter*, since the Bogoliubov transformation is equivalent to a squeezing operation.

On the other hand, the two-particle probability distribution is given by

$$\langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle = \left[ \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_1} \rangle \langle a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_2} \rangle + \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2} \rangle \langle a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_1} \rangle + \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger \rangle \langle a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle \right]. \quad (4)$$

The first term on the r.h.s. is proportional to the product of the two single-inclusive distributions, with momenta  $k_i$ , i.e.,  $N_1(\mathbf{k}_i) = \omega_{\mathbf{k}_i} \frac{d^3 N}{d\mathbf{k}_i} = \omega_{\mathbf{k}_i} \langle a_{\mathbf{k}_i}^\dagger a_{\mathbf{k}_i} \rangle$ , while the second term is proportional to the square modulus of  $G_c(\mathbf{k}_1, \mathbf{k}_2) = G_c(1, 2) = \sqrt{\omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2}} \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2} \rangle$ , the so-called **chaotic amplitude**. The last term is related to this new contribution,  $G_s(\mathbf{k}_1, \mathbf{k}_2) = G_s(1, 2) = \sqrt{\omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2}} \langle a_{\mathbf{k}_1} a_{\mathbf{k}_2} \rangle$ , which is called **squeezed amplitude**.

In cases where the particle is its own anti-particle (for  $\pi^0 \pi^0$  or  $\phi \phi$  boson pairs, for instance), both terms contribute, and the full correlation function is written as

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = 1 + \frac{|G_c(\mathbf{k}_1, \mathbf{k}_2)|^2}{G_c(\mathbf{k}_1, \mathbf{k}_1) G_c(\mathbf{k}_2, \mathbf{k}_2)} + \frac{|G_s(\mathbf{k}_1, \mathbf{k}_2)|^2}{G_c(\mathbf{k}_1, \mathbf{k}_1) G_c(\mathbf{k}_2, \mathbf{k}_2)}, \quad (5)$$

where the first two terms correspond to the HBT correlation, and last term, represents this additional contribution to the correlation function, i.e., the squeezing part.

In the above equations, the amplitudes were written in terms of the asymptotic operators. However, the averages in Eq. (4) are estimated by means of the density operator  $\hat{\rho}$ , as the thermal average for globally thermalized gas of  $b$  quanta that is homogeneous in the system with a certain volume  $V$ , with  $\hat{\rho} = (1/Z) \exp[-(V/T)(1/(2\pi)^3) \int \Omega b^\dagger b]$ . Being so, the above equations should be expressed in terms of the in-medium operators by means of Eq. (3), prior to performing the thermal averages.

We know that the maximum value of the HBT correlation function is attained when  $k_1 = k_2 = k$ , resulting in  $C_c(k, k) = 2$ . Accordingly, the maximum value of the BBC

correlation function is attained for  $k_1 = -k_2 = k$ , resulting, after performing the thermal averages, into

$$C_s(\mathbf{k}, -\mathbf{k}) = 1 + \frac{|c_{\mathbf{k}}^* s_{\mathbf{k}} n_{\mathbf{k}} + c_{-\mathbf{k}}^* s_{-\mathbf{k}} (n_{-\mathbf{k}} + 1)|^2}{n_1(\mathbf{k}) n_1(-\mathbf{k})}, \quad (6)$$

where  $n_1(\mathbf{k}) = [|c_{\mathbf{k}}|^2 n_{\mathbf{k}} + |s_{-\mathbf{k}}|^2 (n_{-\mathbf{k}} + 1)]$  is related to the spectral function by  $N_1(\mathbf{k}) = \frac{V}{(2\pi)^3} \omega_{\mathbf{k}} n_1(\mathbf{k})$ ;  $n_{\mathbf{k}}$  is the Bose-Einstein distribution function of the in-medium quanta with energy  $\Omega_{\mathbf{k}}$  at temperature  $T$ . Strictly speaking Eq. (6) is valid only in the rest frame of the medium.

## 0.2. Finite size medium moving with collective velocity

For a hydrodynamical ensemble, both the chaotic and the squeezed amplitudes,  $G_c$  and  $G_s$ , respectively, can be written in the special form derived by Makhlin and Sinyukov [3] (see Eqs. (22) and (23) of Ref. [1]), namely

$$G_c(\mathbf{k}_1, \mathbf{k}_2) = \int \frac{d^4 \sigma_{\mu}(x)}{(2\pi)^3} K_{1,2}^{\mu} e^{iq_{1,2} \cdot x} \left\{ |c_{1,2}|^2 n_{1,2}(x) + |s_{-1,-2}|^2 [n_{-1,-2}(x) + 1] \right\}, \quad (7)$$

$$G_s(\mathbf{k}_1, \mathbf{k}_2) = \int \frac{d^4 \sigma_{\mu}(x)}{(2\pi)^3} K_{1,2}^{\mu} e^{2iK_{1,2} \cdot x} \left\{ s_{-1,2}^* c_{2,-1} n_{-1,2}(x) + c_{1,-2} s_{-2,1}^* [n_{1,-2}(x) + 1] \right\}. \quad (8)$$

In Eq. (7) and (8),  $d\sigma_{\mu}^4(x) = d^3\Sigma_{\mu}(x, \tau) F(\tau) d\tau$  is the product of the normal-oriented volume element depending parametrically on  $\tau$  (freeze-out hyper-surface parameter) and on its invariant distribution,  $F(\tau)$ ;  $\sigma^{\mu}$  is the hydrodynamical freeze-out surface.

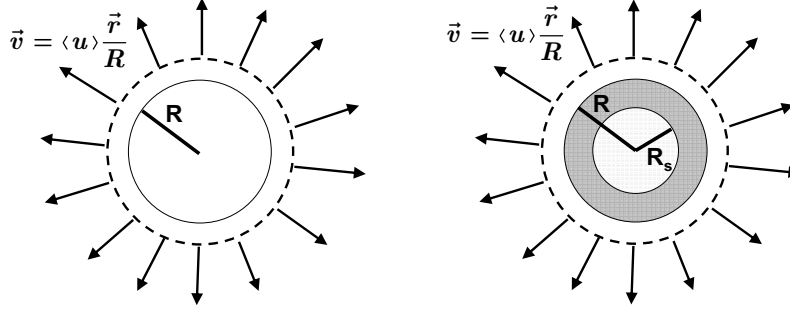
For studying the expansion of the system we adopt the non-relativistic hydrodynamical model of Ref. [4]. In this model the fireball expands in a spherically symmetric manner with a local flow vector given by the four-velocity  $u^{\mu} = \gamma(1, \mathbf{v})$ , assumed to be non-relativistic, with  $\gamma = (1 - \mathbf{v}^2)^{-1/2} \approx 1 + \mathbf{v}^2/2$ , where

$$\mathbf{v} = \langle u \rangle \mathbf{r}/R,$$

being  $\langle u \rangle$  and  $R$  the mean expansion velocity and the radius of the fireball, respectively. We then divide the inhomogeneous medium into independent cells and assume that the expressions for  $G_c$  and  $G_s$  can be evaluated locally within each cell using Eq. (3). The squeezing coefficient can be written, in more detail, as

$$f_{i,j}(x) = \frac{1}{2} \log \left[ \frac{K_{i,j}^{\mu}(x) u_{\mu}(x)}{K_{i,j}^{*\nu}(x) u_{\nu}(x)} \right] = \frac{1}{2} \log \left[ \frac{\omega_{\mathbf{k}_i}(x) + \omega_{\mathbf{k}_j}(x)}{\Omega_{\mathbf{k}_i}(x) + \Omega_{\mathbf{k}_j}(x)} \right] \equiv f_{\pm i, \pm j}(x), \quad (9)$$

where, as in HBT, the pair momentum difference and the pair average momentum are given, respectively, by  $q_{i,j}^{\mu}(x) = k_i^{\mu}(x) - k_j^{\mu}(x)$ , and  $K_{i,j}^{\mu}(x) = \frac{1}{2} [k_i^{\mu}(x) + k_j^{\mu}(x)]$ . In addition, we consider the Boltzmann limit of the Bose-Einstein distribution for  $n_k$ , i.e.,  $n_{i,j} \sim \exp[-(K_{i,j}^{\mu} u_{\mu} - \mu(x))/T(x)]$ , and assume a time-dependent parametric solution of the hydrodynamical equations, i.e.,  $\mu(x)/T(x) = \mu_0/T_0 - r^2/(2R^2)$ , as in Ref. [4].



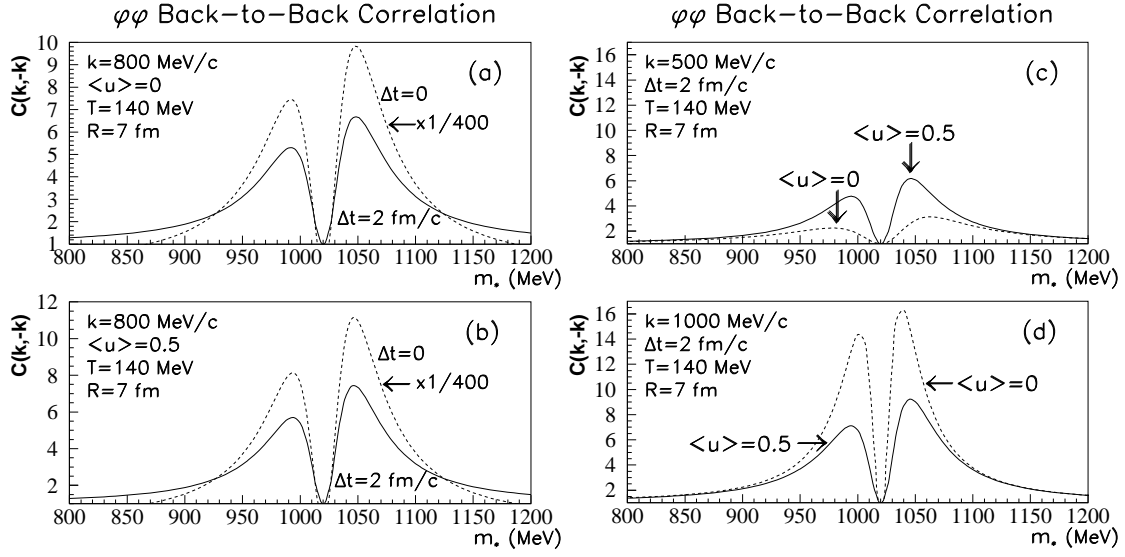
**FIGURE 1.** Schematic representation of the region where the mass-shift occurs: on the l.h.s., the modification is extended to the whole system, whereas on the r.h.s. it happens only in a smaller portion of the thermalized medium. The figures represent cross-sectional areas of the Gaussian profiles.

Furthermore, we consider two possible scenarios for the freeze-out: the first, corresponds to a sudden freeze-out, in which  $F(\tau) \propto \delta(\tau - \tau_0)$ . The second scenario corresponds to a smeared freeze-out, for which  $\frac{\theta(\tau - \tau_0)}{\Delta\tau} e^{-(\tau - \tau_0)/\Delta\tau}$ . This last more realistic scenario has a dramatic effect on the Back-to-Back Correlation function, as already showed in Ref.[1], by reducing severely the signal's magnitude, even for a smearing of about  $\Delta\tau \sim 2$  fm/c.

In discussing finite-size effects, we distinguish between the volume of the entire thermalized medium, denoted by  $V$  (with radius  $R$ ), and the volume filled with mass-shifted quanta, denoted by  $V_s$  (with radius  $R_s$ ). Naturally,  $V_s \leq V$  in the general case. In the derivation of the expressions for  $G_c(1, 2)$  and  $G_s(1, 2)$ , for simplicity, we introduce a Gaussian profile function in the integrands, i.e.,  $\sim \exp[-\mathbf{r}/(2R)^2]$ . In Fig. 1 we illustrate this by showing cross-sectional areas corresponding to Gaussian profiles, for the cases with  $V = V_s$  and  $V > V_s$ .

In the non-relativistic limit, the accounting for the squeezing effects can be simplified for small mass shifts  $(m_* - m)/m \ll 1$ , such that the squeezing parameter in Eq. (9) can be written simply as  $f(i, j, \mathbf{r}) \approx \frac{1}{2} \log\left(\frac{m}{m_*}\right)$ . This limit is important, because in this case the coordinate dependence enters the squeezing parameter  $f$  only through the possible position dependence of the mass-shift which, in principle, could be calculated from thermal field models in the local density approximation. Therefore, in an approximation such that the position dependence of the in-medium mass can be neglected, the  $c(i, j) = c_0$  and  $s(i, j) = s_0$  factors can be removed from the integrands in Eq. (7) and (8) and all what remains to be done are Fourier transforms of Gaussian functions.

For completeness, we write below the expression of the Back-to-Back Correlation function for the case where the mass shift occurs in entire volume of the system,  $V$ . A detailed discussion and more complete formulation of the problem, including the case of mass-shift in a smaller portion of the system, can be found in Ref.[5]. In what follows, we will concentrate on the value of momenta of the participant pair that maximizes the BBC signal, i.e., the case in which  $\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}$ , using the fact that the single-inclusive distribution depends only on the absolute value of the momentum, i.e.,  $G_c(\mathbf{k}, \mathbf{k}) = G_c(-\mathbf{k}, -\mathbf{k})$ . Nevertheless, since the strict condition of back-to-back pair holds only in the rest frame of the medium, we implicitly are considering the case



**FIGURE 2.** In the left panel, parts (a) and (b) illustrate the effect of finite emission times. The dashed curves, corresponding to a sudden emission ( $\Delta t = 0$ ), have been decreased by a factor of 400, and the solid curves show the suppression caused by a finite emission duration, of about  $\Delta t \simeq 2$  fm/c, which drastically reduces the BBC magnitude. In the right panel we illustrate the cases with and without flow, for two values of the momentum of the back-to-back pair (in (c),  $k = 500$  MeV/c and in (d),  $k = 1000$  MeV/c). The mass-shifting is supposed to occur in the entire system volume. In the plots,  $m_*$  is the in-medium modified mass and  $T$  stands for the freeze-out temperature.

corresponding to a weak flow coupling in the expanding system, which is expected to be fulfilled for pair emitted near the center of the system. In this case, the BBC correlation function can then be written as

$$C_{BBC}^V(\mathbf{k}, -\mathbf{k}) = 1 + |c_0 s_0|^2 \left[ 2R^3 \left( 1 + \frac{m^2 \langle u \rangle^2}{m_* T} \right)^{-3/2} \exp \left( -\frac{m_*}{T} - \frac{\mathbf{k}^2}{2m_* T} \right) + R^3 \right]^2 \\ \times \left[ \frac{R^3 (|c_0|^2 + |s_0|^2)}{\left( 1 + \frac{m^2 \langle u \rangle^2}{m_* T} \right)^{3/2}} \exp \left( -\frac{m_*}{T} - \frac{\mathbf{k}^2}{2m_* T} + \frac{m^2 \langle u \rangle^2 \mathbf{k}^2}{2m_*^2 T^2 (1 + \frac{m^2 \langle u \rangle^2}{m_* T})} \right) + R^3 |s_0|^2 \right]^{-2}. \quad (10)$$

In Figure 2 we illustrate some of the results found in this non-relativistic approximation, in the particular case of weak flow coupling. We see that the cases corresponding to the absence of flow and to its inclusion produce similar results within the limits of our illustration. However, depending on the value of  $k_1 = -k_2 = k$ , there are noticeable differences. Being so, we see that, for smaller values of  $k$ , the presence of flow seems to slightly enhance the signal, whereas at large values of  $k$ , the non-flow case wins. Nevertheless, the non-flow case grows faster with increasing  $k$ . In this particular example discussed here, the effect is mainly due to the denominator in Eq. (10), since it contains the flow parameter in the exponential's coefficient.

### 0.3. Summary and Conclusions

Our main goal on presenting these new results here was to revive the discussion on the search of the squeezed BBC. For fulfilling this purpose, we estimated the strength of the BBC signal in a more realistic situation, considering the mass-shifting in a finite region, and the emission occurring during a finite time interval. We also considered that the system expands non-relativistically and analyzed the simplified situation of weak flow dependence (i.e., back-to-back pairs emitted close to central system region) of the BBC. Finally, we employed  $\phi\phi$  pair correlation for illustration. We showed in Figure 2 the back-to-back correlation function versus the in-medium shifted mass,  $m_*$ . We also saw that both the non-flow and the flow cases produced similar results, with pronounced maxima around  $m \approx m_*$ . Although we did not show here results corresponding to the case of mass-shift occurring only in a smaller portion of the system, it is shown elsewhere[5, 6] to decrease proportionally to the size of the mass-shift region. However, the effect of decreasing the system size is far less significant than the finite emission time for reducing the BBC magnitude. We also saw that, at least in its weak-coupling limit, the flow may work for slightly enhance the BBC signal for small values of the momentum  $k$ . Our main conclusion, however, is that, in the particular framework discussed above, a sizeable strength of the squeezed BBC signal could be seen, making it a promising effect to be searched for experimentally at RHIC.

Naturally, for a more refined calculation, it is mandatory to introduce a model-based mass-shift. After that, it is also essential to perform more realistic calculations with flow, in a less particular kinematic region, while simultaneously searching for those which could optimize the observation of the BBC signal. Also an estimate of the shape and width of the BBC around the direction  $k_1 = -k_2 = k$  should be implemented. Finally, for being able to make predictions closer to the experimental conditions, it will be extremely important to obtain some feed-back on the experimental acceptance, conditions, and restrictions that could finally lead to the BBC discovery.

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